

Name:

Math 10a
September 11, 2014
Quiz #1

1. The force between two electrons a distance d apart is

$$F(d) = \frac{C}{d^2}$$

for $d > 0$ and some positive constant C .

- (a) In terms of C , what is the distance as a function of the force?

$$d = \sqrt{\frac{C}{F}}$$

- (b) If the electrons are moving and the scientist measures the force between them going to 0, what can you say about the distance between them?

$$\lim_{F \rightarrow 0} \sqrt{\frac{C}{F}} = +\infty, \text{ i.e., electrons get infinitely far apart}$$

2. Let $f(x) = 10x$, $g(x) = 2^x$.

(a) What is $(g \circ g \circ g \circ f)(.1)$?

$$g(g(g(f(.1)))) = g(g(g(1))) = g(g(2)) = g(4) = \boxed{16}$$

(b) Suppose a population of rabbits doubles every day. Initially (at 0 days) there are 100 rabbits. Write down an expression for the number of rabbits after x days in terms of composing f and g (you may use f and g more than once).

$$R(x) = 100 \cdot 2^x = \boxed{f(f(g(x)))}$$

(c) Same setup as in (b). How many days until there are at least 3000 rabbits?

$$100 \cdot 2^x \geq 3000$$

$$2^x \geq 30$$

$$x \geq \log_2(30)$$

so, after $\log_2(30)$ days (about 5 days).

3. For the following, give the limit or state if it doesn't exist:

(a) $\lim_{x \rightarrow 0} \frac{1}{x^2}$

$+\infty$

(b) $\lim_{x \rightarrow 5} \frac{x^2 - 6x + 5}{x - 5}$.

4

(c) $\lim_{x \rightarrow 0} \frac{1}{1 - 2^x}$.

DNE

($1 - 2^x$ is positive to the left of 0 and negative to the right of 0)

4. Let

$A = \text{range of the function } f(x) = 1 + e^x$

$B = \text{range of the function } g(x) = 4 - x^2.$

What is

(a) $A \cap B$?

$$(-\infty, 4] \cap (1, \infty) = \boxed{(1, 4]}$$

(b) $A \cup B$?

$$(-\infty, 4] \cup (1, \infty) = \mathbb{R}.$$